TYJ – BALLIWALA
MATHEMATICS SOLUTION
DATE : 30-06-2019

31. (b) Given that $\sqrt{-8-6i} = x + iy = z$

$\Rightarrow -8-6i = (x+iy)^2$

$\therefore x^2 - y^2 = -8$ .....(i) and $2xy = -6$ .....(ii)

Now $x^2 + y^2 = \sqrt{64+36} = \pm 10$ .....(iii)

From (i) and (iii), we get $x = \pm 1$ and $y = \pm 3$

Hence $z = \pm(1-3i)$

Trick : Since $\{\pm(1-3i)\}^2 = -8-6i$

32.

33. (c,d) Since $\frac{-\sqrt{3}-i}{2} = -\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\Rightarrow \left(\frac{-\sqrt{3}-i}{2}\right)^3 = -\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3 = -i$

and $\frac{\sqrt{3}-i}{2} = \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$

and $\left(\frac{\sqrt{3}-i}{2}\right)^3 = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$.

Hence the result.

34. (a) $e^{e^{i\theta}} = e^{\cos\theta + i\sin\theta} = e^{\cos\theta}[e^{i\sin\theta}] = e^{\cos\theta}[\cos(\sin\theta) + i\sin(\sin\theta)]$

$\therefore$ Real part of $e^{e^{i\theta}}$ is $e^{\cos\theta}[\cos(\sin\theta)]$

35. (d) Let $z = -1 + i\sqrt{3}$, $r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = \frac{2\pi}{3}$

$\therefore z = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\therefore (z)^{20} = \left[2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]^{20}$

$= 2^{20}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{20} = 2^{20}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{20}$.

36. (d) Given, $\frac{|z-2|}{|z-3|} = 2$

$\Rightarrow \sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$\Rightarrow (x-2)^2 + y^2 = 4[(x-3)^2 + y^2]$

$\Rightarrow x^2 + y^2 + 4 - 4x = 4x^2 + 4y^2 + 36 - 24x$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

or $x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$ .....(i)

We know that, standard equation of circle,

$x^2 + y^2 + 2gx + 2fy + c = 0$ .....(ii)

Comparison of (i) from(ii)

$\Rightarrow 2g = -\frac{20}{3} \Rightarrow g = -\frac{10}{3}, f = 0, c = \frac{32}{3}$

Hence, Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

37. (b) Let $z = x + iy$; $z + iz = (x - y) + i(x + y)$ and $iz = -y + ix$

If A denotes the area of the triangle formed by $z, z + iz$ and iz, then $A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x - y & x + y & 1 \\ -y & x & 1 \end{vmatrix}$

Applying transformation $R_2 \to R_2 - R_1 - R_3$, we get

$$A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 0 \end{vmatrix} = \frac{1}{2}(x^2 + y^2) = \frac{1}{2}|z|^2$$

38. (b) Wehave $\frac{z-1}{z+1} = \frac{x + iy - 1}{x + iy + 1} = \frac{(x^2 + y^2 - 1) + 2iy}{(x+1)^2 + y^2}$

Therefore $arg\,\frac{z-1}{z+1} = \tan^{-1}\frac{2y}{x^2 + y^2 - 1}$

Hence $\tan^{-1}\frac{2y}{x^2 + y^2 - 1} = \frac{\pi}{3}$

$\Rightarrow \frac{2y}{x^2 + y^2 - 1} = \tan\frac{\pi}{3} = \sqrt{3}$

$\Rightarrow x^2 + y^2 - 1 = \frac{2}{\sqrt{3}}y \Rightarrow x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$

Which is obviously a circle.

39. (b) We have $\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2iy}{(1-y)+ix}$

$= \frac{[(2x+1)(1-y) + 2xy] + i[2y(1-y) - x(2x+1)]}{(1-y)^2 + x^2}$

But it is given that imaginary part of $\frac{(2z+1)}{(iz+1)}$ is – 2

$\Rightarrow x + 2y - 2 = 0$. Which is a straight line.

40. (a) $|z - 2 + i| = |z - 3 - i|$

$\Rightarrow |(x-2) + i(y+1)| = |(x-3) + i(y-1)|$

$\Rightarrow \sqrt{(x-2)^2 + (y+1)^2} = \sqrt{(x-3)^2 + (y-1)^2}$

$\Rightarrow x^2 + 4 - 4x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 1 - 2y$

$\Rightarrow 2x + 4y - 5 = 0$.

41. (b) $w = \frac{1 - iz}{z - i}$, then $|w| = 1$

$\Rightarrow \left|\frac{1 - iz}{z - i}\right| = 1 \quad \Rightarrow \quad |1 - iz| = |z - i|$

$\Rightarrow |1 - i(x + iy)| = |x + iy - i|$

$\Rightarrow |(1 + y) - ix| = |x + i(y - 1)|$

$\Rightarrow \sqrt{x^2 + 1 + y^2 + 2y} = \sqrt{x^2 + y^2 + 1 - 2y} \Rightarrow y = 0$

Hence $z = x + iy = x$. So z lies on real axis.

42. (a) $\left|\frac{z}{z - \frac{i}{3}}\right| = 1 \Rightarrow |z| = \left|z - \frac{i}{3}\right|$

Clearly locus of z is perpendicular bisector of line joining points having complex number $0 + i0$ and $0 + \frac{i}{3}$

. Hence z lies on a straight line.

43. (d) $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5} = \frac{(\cos + i\sin\theta)^4}{i^5\left(\frac{1}{i}\sin\theta + \cos\theta\right)^5}$

$= \frac{(\cos\theta + i\sin\theta)^4}{i(\cos\theta - i\sin\theta)^5} = \frac{(\cos\theta + i\sin\theta)^4}{i(\cos\theta + i\sin\theta)^{-5}}$ (By property)

$= \frac{1}{i}(\cos\theta + i\sin\theta)^9 = \sin 9\theta - i\cos 9\theta$.

44. (b) Given that $z = \left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)^5$

$= \left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]^5 + \left[\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right]^5$

$= \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + \cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}$.

Hence Im (z) = 0.

45. (c) $\sin\theta - i\cos\theta = -i^2\sin\theta - i\cos\theta = -i(\cos\theta + i\sin\theta)$

Given expression is

$(-i)^3[\cos(-10\theta - 18\theta + 3\theta) + i\sin(-25\theta)]$

$= i(\cos 25\theta - i\sin 25\theta]$.